IIR filter design by the bilinear transform

The method of filter design by impulse invariance suffers from aliasing. The aliasing will be a problem if the analogue filter prototype's frequency response has significant components at or beyond the Nyquist frequency.

The problem with which we are faced is to transform the analogue filter design into the sampled data z plane Argand diagram. The problem of aliasing arises because the frequency axis in the sampled data z plane Argand diagram is a circle:

- In the analogue domain the frequency axis is an infinitely long straight line
- In the sampled data z plane Argand diagram the frequency axis is a circle

![Diagram showing the transformation from the infinite, straight analogue frequency axis to the finite, circular digital frequency axis.](image)

The analogue filter must be small at the aliasing frequency.

Note also that:
- In the analogue domain transient response is shown along the horizontal axis
- In the sampled data z plane Argand diagram transient response is shown radially outwards from the centre

The problem of aliasing arises because we wrap an infinitely long, straight frequency axis around a circle. So the frequency axis wraps around and around, and any components above the Nyquist frequency get wrapped back on top of other components.

![Diagram showing the bilinear transform.](image)

The bilinear transform is a method of squashing the infinite, straight analogue frequency axis so that it becomes finite. This is like squashing a concertina or accordion. To avoid squashing the filter's desired frequency response too much, the bilinear transform squashes the far ends of the frequency axis the most - leaving the middle portion relatively unsquashed. The infinite, straight analogue frequency axis is squashed so that it becomes finite - in fact just long enough to wrap around the unit circle once only. This is also sometimes called frequency warping.